Behavioural analysis of urea decomposition system in a fertiliser plant

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Abstract: Reliability, availability and maintainability (RAM) analysis of a system is helpful in carrying out design modifications, if any, and it is required to achieve minimum failures or to increase mean time between failures (MTBF), and thus it is used to plan maintainability requirements, optimise reliability and maximum equipment availability. To this effect, this paper presents the application of RAM analysis in the process industry. Fuzzy Lambda–Tau methodology is used here to model the system behaviour. Each system components, failure rate and repair time are represented by triangular fuzzy numbers with known spread. The methodology uses Petri nets to model the system instead of fault tree because it allows efficient simultaneous generation of minimal cuts and path sets. Various reliability parameters such as MTBF, expected number of failures (ENOF), reliability, availability, etc. are calculated. The computed results are presented to plant personnel for their performance considerably by adopting and practicing suitable maintenance policies/strategies.

Keywords: reliability; availability; maintainability; PNs; Petri nets; Lambda–Tau methodology; fuzzy sets.

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1 Introduction

Reliability, in general, can be defined as the ability of a system to perform its required functions under stated conditions for a specified period of time. Reliability technology is an important phenomenon of the existing era. This technology is widely used to increase the efficiency of the system. To overcome day to day problems, nowadays, the system analysts and engineers are interested in the analysis of the reliability models to implement them for practical utility. In a fertiliser industry (a process industry), for the production of urea, it has to go through many processes, that is, synthesis, decomposition, crystallisation, prilling and recovery. One of the important functionary parts of this industry is the decomposition process. In this paper, a method of calculating availability, reliability, MTBF, repair time, etc. for a decomposition process is discussed. The presented system consists of five subsystems, with a standby unit in one of the subsystems. This paper also deals the case when no standby unit is taken in the process. Behaviour analysis of each item of equipment under given operating conditions is helpful to design the component for minimum failure and to prepare a plan in advance for scheduled maintenance or preventive maintenance.

To remain competitive and to provide timely and accurate services, the companies are viewing reliability and maintainability issues as a part of corporate quest for improving quality of the products/processes and services delivered. A company cannot adopt a rapid response strategy if its systems are unavailable and unreliable. The management is highly concerned with reliable operation of production systems. To this effect and to plan/ adopt suitable maintenance strategies behavioural knowledge of the system(s), their component(s) is customary. These challenges imply that a new and pragmatic approach towards the reliability, availability and maintainability (RAM) of production systems in unison must be adopted because organisational performance and survivability hinges a lot on reliability and maintainability of its components/parts and system(s) as a whole. Among the various tools of technology for performance modelling (root cause analysis (RCA); failure mode and effect analysis (FMEA); RAM), RAM an engineering tool evaluates the equipment performance at different stages in design process. With RAM analysis of the system key performance metrics such as mean time to failure (MTTF), Equipment down time (EDT) and system availability values (A_{sys}) can be ascertained. The information obtained from analysis helps the management in assessment of the RAM needs of system.

This paper is concerned with behaviour analysis of urea decomposition system in fertiliser plant. This paper is organised as follows. Section 2 deals with the literature review. Section 3 contains the mathematical aspects of the RAM analysis. In Section 4, the methodology used for determining the expressions of different reliability parameters for a general repairable system is discussed with sections containing basics of Petri nets (PNs), modelling with PNs and definitions of minimal cut and path sets. Section 5 is devoted to the fundamentals of the fuzzy set theory. Section 6 contains the system description along with the RAM analysis of the considered system with and without standby unit. Finally, some concrete conclusions have been presented in Section 7.

2 Literature review

The majorities of industrial systems are repairable and consist of several subsystems. Each subsystem is composed of numerous complex components and the probability that the system survives depends directly on each of its constituent components. So, to remain competitive and to provide timely and accurate services, the companies view the reliability and maintainability issues as a part of corporate quest to improve quality of the products/processes and services delivered. A company cannot adopt a rapid response strategy if its systems are unavailable and unreliable. However, failure is an unavoidable phenomenon in mechanical systems/process plants/components. These failures may be the result of human error, poor maintenance or inadequate testing and inspection.

Therefore, the systems and components undergo several failure-repair cycles that include logistic delays while performing repairs and it leads to the degradation of systems' overall performance. Behaviour of these systems will help to analyse the systems' overall performance and to carryout design modifications. But there is no doubt that the success of these will mainly depend upon the reliable operation of production systems. The complexity of industrial systems and the non-linearity of their behaviour are such that the explicit functions, modelling the system evolution, are not readily available. In general, RAM and their composite measures are used to measure the behaviour/ performance of these systems. Numbers of techniques have been used for this purpose. Some widely used techniques are event tree, fault tree analysis (FTA), reliability block diagrams (RBDs), PNs, Markovian approach, etc. (Adamyan and David, 2002; Ebeling, 2001). Rooney and Turner (1988) conducted a preliminary hazard analysis on actual fluid catalytic cracking units in chemical refineries using fault tree representations and suggested qualitative recommendations for improving availability. McCluer and Whittle (1992) in their work reviewed three petroleum refineries with RBDs to identify potential effects of single failures. Kumar (1991) and Kumar and Pandey (1993) used the Markovian approach in their analysis and evaluated the performance of sugar, paper and urea fertiliser plants in terms of availability. Arora and Kumar (1997) have done availability analysis of steam and power generation systems in the thermal power plant. Again, Arora and Kumar (2000) have done behaviour analysis and maintenance management for coal handling system in a thermal power plant. Gupta et al. (2005, 2007) and Kumar (2009) evaluated reliability parameters of a butter-oil (melted butter) manufacturing plant and plastic-pipe manufacturing plant, respectively. Reliability, availability and MTBF of the subsystems have been computed for various choices of failure and repair rates of subsystems of the plants. Zio et al. (2006) have done the availability evaluation of a multi-state, multi-output offshore installation using Monte Carlo simulation. Aksu et al. (2006) proposed reliability assessment methodology using failure mode and effect analysis (FMEA), FTA and Markovian approach complementarily; and described its applications in the reliability and availability assessment of pod propulsion system. Wattanapongskorn and Coit (2007) modelled embedded system design and optimisation, considering component redundancy and uncertainty in the component reliability estimates. Sharma et al. (2005) had discussed the behavioural analysis and resources allocation of an industrial system by using FMEA.

Sachdeva et al. (2008a,b) have done reliability analysis of the pulping system in a paper mill, a complex repairable industrial system, using PN. They proposed a methodology based on PN to evaluate the reliability characteristics of the pulping system of a paper industry in a realistic working environment. Again, Sachdeva et al. (2008a,b) did availability modelling of the screening system using generalised stochastic PN in a paper mill. Khan et al. (2008) have presented a risk-based methodology to estimate optimal inspection and maintenance intervals which maximise the systems availability. Aldaihani and Savsar (2008) developed a stochastic model to analyse performance measures of a flexible manufacturing cell (FMC) consisting of two machines served by a robot for loading and unloading purposes, and a pallet handling system, under different operational conditions, including machine failures and repairs based on Markov processes. Korayem and Iravani (2008) improve the reliability and quality of 3P and 6R mechanical robots by using applied FMEA and quality function deployment (QFD) approach. Sharma and Kumar (2008) presented the application of RAM analysis in a process industry using Markovian approach. Komal et al. (2010) proposed the GABLT technique to predict the behaviour analysis of an industrial system.

Wu (2004, 2006), Huang et al. (2006) and Viertl (2009) performed Bayesian system reliability assessment under fuzzy environments. Azaron et al. (2009) solved a multi-objective discrete reliability optimisation problem in a *k*-dissimilar-unit with non-repairable cold-standby redundant system using genetic algorithm approach. Zio et al. (2009) proposed an approach for assessment of dependence among human errors, an important aspect of human reliability analysis.

All the above-examined systems are repairable ones and data used for their behaviour analysis were collected from historical records/logbooks/experts opinions and taken as crisp data because most of the industrial systems exhibit constant failure and repair rates after initial burn-in period. Since records are either not properly updated or out of date, so did not reflect the actual behaviour of the system. Thus, the used data were vague, imprecise and uncertain, that is, historical records can only represent the past behaviour but may be unable to predict future behaviour of the equipment. Also, the traditional analytical techniques need large amounts of data, which are difficult to obtain because of various practical constraints such as rare events of components, human errors and economic considerations for the estimation of failure/repair characteristics of the system. In such circumstances, it is usually not easy to analyse the behaviour and performance of these systems up to desired degree of accuracy by utilising available resources, data and information. Furthermore, if analysis has been done by using some suitable techniques listed above, then any reliability index alone is inadequate to give deeper idea about such type of systems' behaviour because a lot of factors exist which overall influence the systems' performance and consequently their behaviour. Thus, to analyse more closely the system's behaviour, other reliability criteria should be included in the traditional analysis and involved uncertainties must be quantified. The inclusion of various reliability indices as criteria helps the management to understand the effect of increasing/decreasing the failure and repair rates of a particular component or subsystem upon the overall performance of the system and quantification of uncertainties provide results closer to the real-situational environment's results.

Knezevic and Odoom (2001) highlighted these ideas and analysed the behaviour of a general repairable system by introducing the concept of fuzzy Lambda–Tau technique with PN in terms of various reliability indices utilising quantified data. In their approach, PN is used to model the system while fuzzy set theory is used to quantify the uncertain, vague and imprecise data. The use of fuzzy set theory (Zimmermann, 1996) and fuzzy arithmetic to determine components or system reliability can be found in literature (Cai, 1996; Cai et al., 1991; Savoia, 2002). Knezevic and Odoom (2001) used fuzzy triangular numbers to quantify the involved uncertainties in the failure/repair data because it is easy for preparation, evaluation and interpretation of engineering data. In their analysis, several reliability indices are used such as failure rate, repair time, mean time between failures (MTBF), expected number of failures (ENOF) and availability and reliability of the system which gave more sound idea about the system's behaviour. Komal et al. (2007) used this approach for behaviour analysis of the feeding system using FTA instead of PN, Sharma et al. (2007) for the pulping system using PN, in a paper mill. Sharma et al. (2008, 2009) analysed the reliability of multi-robotic system and computed various reliability parameters using fuzzy approach.

Thus, the main objective of this paper is to quantify the uncertainties with the help of fuzzy numbers and to develop a technique to analyse the system's behaviour more closely and to make the decisions more realistic and generic for further application. In this paper, fuzzy Lambda–Tau methodology (FLTM) has been developed for behaviour analysis of complex repairable industrial systems. Thus, it is observed from the study that using uncertain and limited data for complex repairable industrial system, stochastic behaviour can be analysed up to a desired degree of accuracy. Plant personnel may use the results and can give guidelines to improve the system's performance by adopting suitable maintenance strategies. An example of the urea plant (situated in northern part of India) is taken into account to demonstrate the proposed technique. The obtained results will help the management for reallocating the resources to achieve the targeted goal of higher profit.

3 Mathematical aspects of RAM analysis

This section is devoted to the mathematical aspects for RAM analysis for the system. The study described is valid when the failure rate of the system is considered to be constant. The following are the mathematical terms which are interrelated to the desired work (Ebeling, 2001).

3.1 Reliability

It is a measure of the probability for failure-free operation of a system during a given interval under specified operating conditions, that is, it is a measure of success for a failure-free operation. The reliability of a component is calculated as

$$R(t) = 1 - e^{-\lambda t}$$

where λ is the constant failure rate of the component in per hour and t is the operational time in hours.

3.2 Availability

Barlow defined availability as the probability that a component or system is performing its required function at a given point in time when used under stated operating conditions.

Availability can be divided into three categories:

- *Instantaneous availability*: point or system (or component) will be operational at any random time t. This is very similar to the reliability function in that it gives a probability that a system will function at the given time t. Unlike reliability, the instantaneous availability measure incorporates maintainability information. At any given time t, the system will be operational if the following conditions are met:

$$A_s(t) = \frac{\mu_s}{\mu_s + \lambda_s} + \frac{\lambda_s}{\mu_s + \lambda_s} e^{-(\lambda_s + \mu_s)t}$$

- *Average availability*: it is defined on an interval of the real line. If we consider an arbitrary interval [*a b*], then average availability is represented as

$$A_v = \frac{1}{b-a} \int_a^b A_s(t)\,\mathrm{d}t$$

- *Steady state availability*: the steady state availability of the system is the limit of the instantaneous availability function as time approaches to infinity. The instantaneous availability function approaches the steady state value very closely at time that approximates to four times the MTBF. Limiting (or steady-state) availability is represented by

$$A = \lim_{t\to\infty} A_s(t) = \frac{\text{MTBF}}{\text{MTBF} + \text{MTTR}}$$

3.3 Maintainability

It deals with duration of maintenance outages or *how long* it takes to complete the maintenance actions. Maintainability characteristics are usually determined by equipment design, which then sets maintenance procedures and determine the length of repair times. A key maintainability figure of merit is the mean time to repair (MTTR) and a limit for maximum repair time. Qualitatively, it refers to the case with which hardware or software is restored to a functioning state. Quantitatively, it has probabilities and its measure is based on the total down time for maintenance including all time for diagnosis, trouble shooting, tear-down, removal/replacement, active repair time, verification testing that the repair is adequate, delays for logistic movements and administrative maintenance delays.

It is often expressed as

$$M(t) = 1 - \exp\left(\frac{-t}{\text{MTTR}}\right) = 1 - e^{-\mu t}$$

where μ is constant maintenance rate and MTTR is the mean time to repair.

4 Lambda–Tau methodology

Lambda–Tau methodology is a traditional method in which fault tree is used to model the system. The constant failure rate model is adopted in this method and the basic expressions used to evaluate systems failure rate (λ) and repair time (τ) associated with the logical AND and OR-gates (Dhillion and Singh, 1991) are given in Table 2. Knezevic and Odoom (2001) extended this idea by coupling it with PN. Their approach is based on qualitative modelling using PN and quantitative modelling analysis using Lambda–Tau method of solution with basic events represented by fuzzy numbers of triangular membership function. They computed various reliability indices for behaviour analysis of a general repairable system. Their approach is limited and because of that, as the number of components of the system increases or system structure becomes more complex, the computed reliability indices in the form of fuzzy membership function have wide spread due to various fuzzy arithmetic operations used in the calculations (Chen, 1994) (Table 3). It means these indices have high range of uncertainty and thus cannot give the exact idea about the system's behaviour and consequently its performance. Thus, this approach is not suitable for the behaviour analysis of large and complex repairable industrial systems when data are imprecise and represented by fuzzy numbers. To analyse the behaviour of a complex industrial system stochastically up to a desired degree of accuracy, an effective technique is needed. Technique should reduce the uncertainty level, that is, spread of each reliability index must be reduced so that plant personnel may use these indices to analyse the systems behaviour in more promising way.

The Lambda–Tau method incorporates many other restrictions. The main restrictions on its application are

- the ratio of the each basic event repair time, τ, to the mission time, T, must be small
- the basic event failure rates, λ, are very small
- the product of τ and λ is very small
- the product of λ and T is very small
- τ and λ must be constant (i.e. the negative exponential distribution must be applied in the quantitative analysis), failure occurs independently
- after repairs, the repaired component is considered as good as new.

4.1 PNs theory

Petri (1962) proposed PN for modelling the dynamic behaviour of sequential asynchronous automatons (Murata, 1989). Mathematically, PN is a five-tuple,

$$\text{PN} = (P, T, F, W, M_0)$$

where $P = \{p_1, p_2, \ldots, p_m\}$ is a finite set of places; $T = \{t_1, t_2, \ldots, t_n\}$ is a finite set of transitions; $F \subseteq (P \times T) \cup (T \times P)$ is a set of arcs (flow relation); W: $F \rightarrow \{1, 2, 3, \ldots\}$ is a weight function and M_0: $P \rightarrow \{0, 1, 2, \ldots\}$ is the initial marking.

$$P \cap T = \varphi$$

and

$$P \cup T \neq \varphi$$

PN with given initial marking is represented as (N, M_0). The classical PNs are useful in investigating the qualitative or logical properties of concurrent systems, such as mutual exclusion, existence and absence of deadlocks, boundedness, etc. However, for quantitative performance evaluation, the concept of time needs to be added to the definition of PNs.

In the field of reliability engineering, it is very easy to understand and apply PN. Modelling through PN is basically a graphical method utilising some basic symbols for describing relations between the events and conditions. PN has a static as well as dynamic part. The static part is made up of the three objects: places, transitions and arrows. The dynamic part is marking of the graph and it is made up of the various tokens, which are present or not present in the various places and evolves dynamically according to the firing of the various valid transitions.

In PN model, places (events) correspond to discrete states represented by circle while the transitions (conditions) are represented by bars. The execution of the PN is controlled by the position of the token. The firing of transitions moves tokens. However, for a token to be moved, a transition must be enabled. During firing, the enabling tokens are removed from the input places and new tokens are generated at the output places (Knezevic and Odoom, 2001; Murata, 1989).

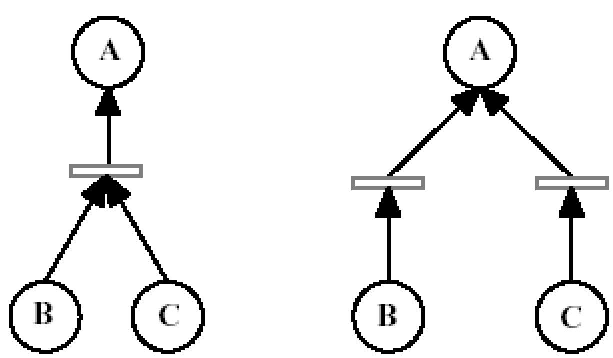
PN can also be used to analyse the dynamic behaviour of systems and it is most useful in modelling state transitions in complex system because they provide an easy way to understand the model of information flow.

4.2 System modelling with PNs

Similar to fault tree model, PN also represents graphically the cause and effect relationship and interaction among the working units of the system to be modelled (Liu and Chiou, 1997). In the graphical model, places and transitions are connected by arcs. A directed arc from a transition to a place is said to be an input arc and the one from place to transition is called an output arc, with respect to the place and vice versa with respect to the transition. The number of places as well as number of transitions are finite but not zero (Murata, 1989).

In this study, only the static part of PN is used. That is, the tokens are omitted and it is assumed that transitions are not timed, that is they are immediate transitions. Figure 1(a) and (b) illustrate the equivalent PN models, corresponding to the logical basic AND and OR gates, respectively.

Figure 1 PN models of logical (a) AND and (b) OR gates



(a) (b)

4.3 Minimal cut and path sets using PN model

A cut set is a set of components whose failure will result in a system failure and a minimal cut set is one in which all the component must fail in order that the system fails. Similarly, a path set is a set of components whose functioning ensures that the system functions and a minimal path set is one in which all the components within the set must function for the system to function. Minimal cut and path sets can be derived from a PN model more efficiently than from an equivalent fault tree model. It was demonstrated through a matrix method (Liu and Chinu, 1997) that the determination of minimal path sets could be achieved in PN model without transforming the PN to its dual and thus ensured that the minimal cut and path sets can be determined at the same time.

5 Fundamental of fuzzy sets and membership functions

In classical sets, the membership of any object is precisely defined whereas the fuzzy sets contain objects that satisfy imprecise properties of membership; that is membership of an object in a fuzzy set can be partial. For classical sets, an element *x* in a universe *U* is either a member of some crisp set *A* or it is not.

This is represented mathematically by the indicator function

$$X_A(x) = \begin{cases} 1, & x \in A \\ 0, & x \notin A \end{cases}$$

Zadeh (1975) extended the notion of binary membership to accommodate various degrees of membership on the real-continuous interval [0, 1], where the endpoints of 0 and 1 conform to no membership and full membership, respectively. Just as the indicator function does for crisp sets, the infinite number of values in between the end points can represent various degrees of membership for an element *x* in some set of universe *U*. The sets of the universe *U* that can accommodate degrees of membership were termed by Zadeh as fuzzy sets. Hence, a fuzzy set can be represented by a functional mapping as

$$\mu_{\tilde{A}}(x) \in \tilde{A} \tag{1}$$

where $\mu_{\tilde{A}}(x)$ is the degree of membership of element *x* in fuzzy set $\tilde{A}$ or simply a membership function of $\tilde{A}$. The value $\mu_{\tilde{A}}(x)$ is on the unit interval that measures the degree to which element *x* belongs to fuzzy set $\tilde{A}$; equivalently, we write

$$\mu_{\tilde{A}}(x) = \text{ the degree to which } x \in \tilde{A}$$

The larger $\mu_{\tilde{A}}(x)$ is, the stronger is the degree of belongingness for *x* in $\tilde{A}$.

5.1 Extension principle

This principle is used to extend mathematical laws of crisp numbers to fuzzy numbers. The extension principle was developed by Zadeh (1975) and later elaborated by Yager (1986) to enable the extension of the domain of a function on fuzzy sets. It plays a fundamental role in translating the set-based concepts to fuzzy set counterparts. A principle for fuzzifying crisp functions (or possibly crisp relations) is called an extension principle (Klir and Yuan, 2005; Zimmermann, 2001). We say a crisp function

$$f : X \to Y$$

is fuzzified when it is extended to act on fuzzy sets $\tilde{F}(X)$ and $\tilde{F}(Y)$, defined on *X* and *Y*. The corresponding fuzzified function, usually denoted by the same symbol *f*, has the form,

$$f : \tilde{F}(X) \to \tilde{F}(Y)$$

And its inverse function f^{-1} has the form

$$f^{-1} : \tilde{F}(Y) \to \tilde{F}(X)$$

5.2 Alpha(α)-cuts

An α -cut of a fuzzy set $\tilde{A}$ is a crisp set consisting of elements of $\tilde{A}$ at least to a degree α . α -cut of a fuzzy set $\tilde{A}$ is denoted by A^{α} and mathematically defined by

$$A^{\alpha} = \left\{ x \in U : \mu_{\tilde{A}}(x) \geq \alpha \right\} \tag{2}$$

where α is a parameter in a range $0 \leq \alpha \leq 1$ and *U* is the universe of discourse. The concept of α-cuts offers a method for resolving any fuzzy sets in terms of constituent crisp sets.

5.3 Membership function and interval analysis

The concept of membership function is most important in fuzzy set theory. Membership functions are used to represent various fuzzy sets. Many different membership functions such as triangular, trapezoidal, normal, gamma, etc. can be used to represent fuzzy sets. However, triangular and trapezoidal fuzzy numbers are mostly used in reliability calculations as they can easily model the imprecise information such as about five, high reliability, low-failure rate, etc. They are also easy for preparation, evaluation and interpretation of engineering data. A triangular fuzzy number is represented by triplet (*a, b, c*) and defined by membership function $\mu_{\tilde{A}}(x) : R \to [0,1]$ is equal to

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a}{b-a} & \text{if } a \leq x \leq b \\ 1 & \text{if } x = b \\ \frac{c-x}{c-b} & \text{if } b \leq x \leq c \end{cases} \tag{3}$$

The α-cut of a triangular fuzzy numbers (*a*, *b*, *c*) is defined below and shown in Figure 2.

$$A^{(\alpha)} = \left[a^{(\alpha)}, c^{(\alpha)} \right] \tag{4}$$

The interval of confidence defined by α-cuts can be written as:

$$A^{(\alpha)} = [(b-a)\alpha + a, -(c-b)\alpha + c] \tag{5}$$

The basic arithmetic operations, that is, addition, subtraction, multiplication and division based on two fuzzy sets $\tilde{A}$ and $\tilde{B}$ are shown in Table 1 using the following definitions:

$$A^{\alpha} = \left[A_1^{\alpha}, A_3^{\alpha}\right], \quad B^{\alpha} = \left[B_1^{\alpha}, B_3^{\alpha}\right], \quad \alpha \in [0,1]$$

It is clear that the multiplication and division of two triangular fuzzy numbers are not triangular fuzzy number with linear sides but are new fuzzy numbers parabolic sides. Now using the above basic discussion, the logical AND and OR-transitions of PN model can be formulated.

The basic expressions of FLTM and reliability parameters for the system are given in Tables 2 and 3, respectively.

Figure 2 A triangular membership function with α-cuts

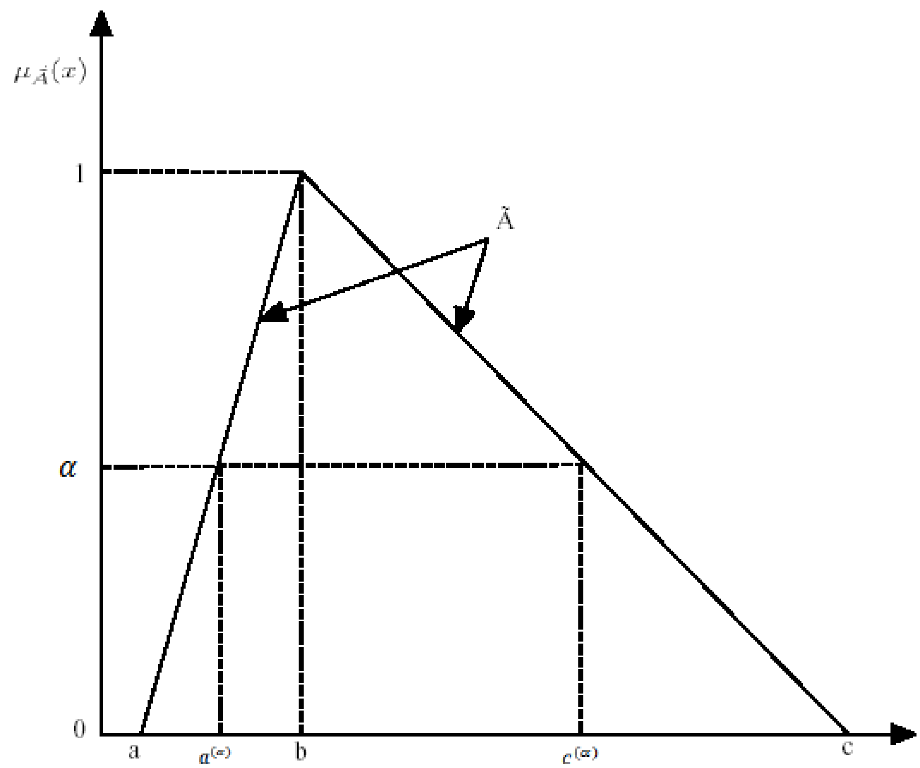


Table 1 Basic operations on fuzzy numbers

Operation	*Crisp*	*Fuzzy*
Addition	$A+B$	$\tilde{A}+\tilde{B}=\left[A_1^{(\alpha)}+B_1^{(\alpha)}, A_3^{(\alpha)}+B_3^{(\alpha)}\right]$
Subtraction	$A-B$	$\tilde{A}-\tilde{B}=\left[A_1^{(\alpha)}-B_3^{(\alpha)}, A_3^{(\alpha)}-B_1^{(\alpha)}\right]$
Multiplication	$A\cdot B$	$\tilde{A}\cdot\tilde{B}=\left[A_1^{(\alpha)}\cdot B_1^{(\alpha)}, A_3^{(\alpha)}\cdot B_3^{(\alpha)}\right]$
Division	$A\div B$	$\tilde{A}\div\tilde{B}=\left[A_1^{(\alpha)}\div B_3^{(\alpha)}, A_3^{(\alpha)}\div B_1^{(\alpha)}\right]$, if $0\notin\left[B_1^{(\alpha)}, B_3^{(\alpha)}\right]$

Table 2 Basic expression of $\lambda-\tau$ methodology

Gate $\rightarrow$	λ_{AND}	τ_{AND}	λ_{OR}	τ_{OR}
Expressions	$\prod_{j=1}^{n}\lambda_j\left[\sum_{i=1}^{n}\prod_{\substack{j=1\\ i\neq j}}^{n}\tau_j\right]$	$\dfrac{\prod_{i=1}^{n}\tau_i}{\sum_{j=1}^{n}\left[\prod_{\substack{i=1\\ i\neq j}}^{n}\tau_i\right]}$	$\sum_{i=1}^{n}\lambda_i$	$\dfrac{\sum_{i=1}^{n}\lambda_i\tau_i}{\sum_{i=1}^{n}\lambda_i}$

Table 3 Some Reliability Parameters

Parameters	*Expressions*
Mean time to failure	$\text{MTTF}_s = \dfrac{1}{\lambda_s}$
Mean time to repair	$\text{MTTR}_s = \dfrac{1}{\mu_s} = \tau_s$
Expected number of failures	$W_s(0,t) = \dfrac{\lambda_s\mu_s t}{\lambda_s+\mu_s} + \dfrac{\lambda_s^2}{(\lambda_s+\mu_s)^2}\left[1-e^{-(\lambda_s+\mu_s)t}\right]$
Mean time between failures	$\text{MTBF}_s = \text{MTTF}_s + \text{MTTR}_s$
Availability	$A_s = \dfrac{\mu_s}{\lambda_s+\mu_s} + \dfrac{\lambda_s}{\lambda_s+\mu_s}e^{-(\lambda_s+\mu_s)t}$
Reliability	$R_s = e^{-\lambda_s t}$
Maintainability	$M_s = 1-e^{-\mu_s t}$

The interval expressions for the fuzzy numbers with triangular membership function for AND/OR-transitions are as follows:

Expression for AND transition:

$$\lambda^{(\alpha)} = \left[\prod_{i=1}^{n}\{(\lambda_{i2}-\lambda_{i1})\alpha+\lambda_{i1}\}\sum_{j=1}^{n}\left[\prod_{\substack{i=1\\ i\neq j}}^{n}\{(\tau_{i2}-\tau_{i1})\alpha+\tau_{i1}\}\right],\ \prod_{i=1}^{n}\{-(\lambda_{i3}-\lambda_{i2})\alpha+\lambda_{i3}\}\sum_{j=1}^{n}\left[\prod_{\substack{i=1\\ i\neq j}}^{n}\{-(\tau_{i3}-\tau_{i2})\alpha+\tau_{i3}\}\right]\right] \tag{6}$$

$$\tau^{(\alpha)} = \left[\frac{\prod_{i=1}^{n}\{(\tau_{i2}-\tau_{i1})\alpha+\tau_{i1}\}}{\sum_{j=1}^{n}\left[\prod_{\substack{i=1\\ i\neq j}}^{n}\{-(\tau_{i3}-\tau_{i2})\alpha+\tau_{i3}\}\right]},\ \frac{\prod_{i=1}^{n}\{-(\tau_{i3}-\tau_{i2})\alpha+\tau_{i3}\}}{\sum_{j=1}^{n}\left[\prod_{\substack{i=1\\ i\neq j}}^{n}\{(\tau_{i2}-\tau_{i1})\alpha+\tau_{i1}\}\right]}\right] \tag{7}$$

Expression for OR transition:

$$\lambda^{(\alpha)} = \left[\sum_{i=1}^{n} \left\{ \left(\lambda_{i2} - \lambda_{i1} \right) \alpha + \lambda_{i1} \right\}, \quad \sum_{i=1}^{n} \left\{ -\left(\lambda_{i3} - \lambda_{i2} \right) \alpha + \lambda_{i3} \right\} \right] \tag{8}$$

$$\tau^{(\alpha)} = \left[\frac{\sum_{i=1}^{n} \left[\left\{ \left(\lambda_{i2} - \lambda_{i1} \right) \alpha + \lambda_{i1} \right\} \left\{ \left(\tau_{i2} - \tau_{i1} \right) \alpha + \tau_{i1} \right\} \right]}{\sum_{i=1}^{n} \left\{ -\left(\lambda_{i3} - \lambda_{i2} \right) \alpha + \lambda_{i3} \right\}}, \frac{\sum_{i=1}^{n} \left[\left\{ -\left(\lambda_{i3} - \lambda_{i2} \right) \alpha + \lambda_{i3} \right\} \left\{ -\left(\tau_{i3} - \tau_{i2} \right) \alpha + \tau_{i3} \right\} \right]}{\sum_{i=1}^{n} \left\{ \left(\lambda_{i2} - \lambda_{i1} \right) \alpha + \lambda_{i1} \right\}} \right] \tag{9}$$

5.4 Defuzzification

As most of the actions or decisions implemented by humans or machines are binary or crisp so it is necessary to convert the fuzzy output to a crisp value. The process of converting fuzzy output to a crisp value is said to be *defuzzification*. There exist many defuzzification techniques in the literature such as max-membership principle, centre of area COA, centre of sum, centre of largest area etc., which can be used depending on the application (Klir and Yuan, 2005; Ross, 2004; Zimmermann, 2001). The COA method is selected for this study as it is equivalent to mean of data and so it is very appropriate for reliability calculation. If the membership function $\mu_{\tilde{A}}(x)$ of the output fuzzy set $\tilde{A}$ is described on the interval $\left[x_1, x_2 \right]$, then COA defuzzification $\overline{x}$ can be defined as:

$$\overline{x} = \frac{\int_{x_1}^{x_2} x \mu_{\tilde{A}}(x) \, \mathrm{d}x}{\int_{x_1}^{x_2} \mu_{\tilde{A}}(x) \, \mathrm{d}x} \tag{10}$$

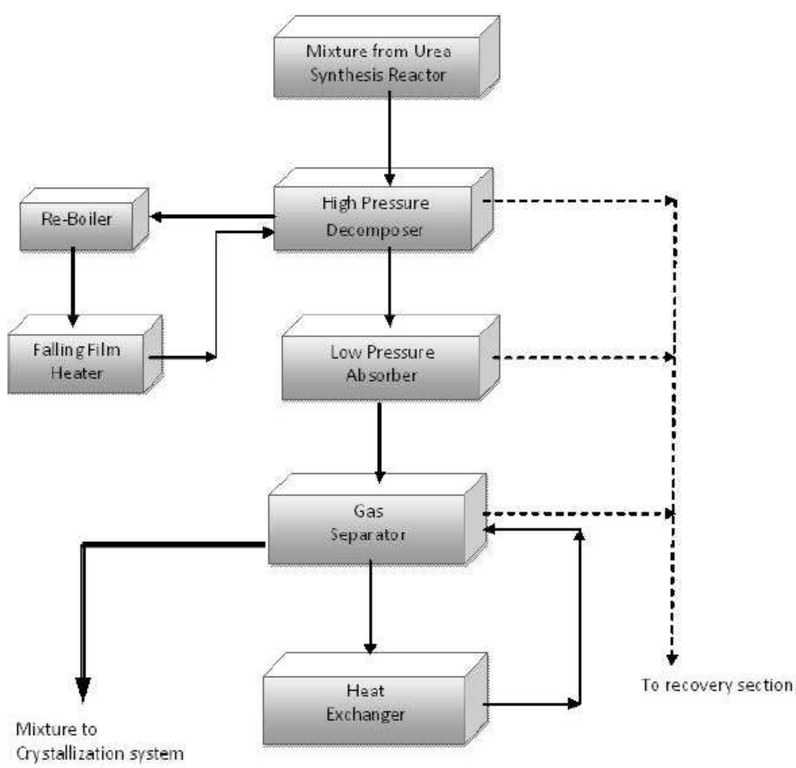
6 System description and RAM analysis

The urea plant considered here is a complex engineering system where the units are arranged in a random fashion and they run continuously for a longer period to produce the required quantity of urea. The plant is a combination of two dependent systems namely ammonia production system and the urea production system. For the production of urea, liquid ammonia and carbon dioxide are used as inputs which are obtained from ammonia plant. Further, processed in a reactor at controlled pressure and temperature the reactants (urea, ammonium carbonate, water and excess ammonia) are sent to decomposer for urea separation. In the crystalliser, the crystals of urea are separated by centrifuge and conveyed pneumatically to the prilling tower where they are melted, sprayed through distributors and finally; fell down at the bottom of the tower, from where it is collected. Among the various functional units in the plant such as urea synthesis, urea decomposition, urea crystallisation, urea prilling and urea recovery, urea decomposition, one of the most important and vital functional processes is the subject of discussion in RAM analysis. The schematic diagram of the system is shown in Figure 3.

In brief, the various subsystems and the components associated with them are defined as below:

- Subsystem 1 [SS_1] has two units in series, that is, A_1 and A_2 where A_1 is called reboiler (the component used to reboil the mixture of liquid ammonia + CO_2 + biuret and urea obtained from high-pressure decomposer at a temperature of about 151 °C and A_2 is called falling film heater (used to heat the mixture to about 165 °C).
- Subsystem 2 [SS_2] has two units in series, that is, B_1 and B_2 where B_1 is called high-pressure absorber and B_2 is called low-pressure absorber (the component contained of sieve trays and packed bed for tripping off the remaining ammonia). Failure of either unit causes the failure of whole system.
- Subsystem 3 [SS_3], the gas separator (C), has one unit only (used to separate the gases obtained from pressure absorbers. The solution is fed to crystallisation unit for further processing).
- Subsystem 4 [SS_4] has two units in series, that is, D and E where D is low-pressure heat exchanger and E is high-pressure heat exchanger with standby unit (the heat exchangers are used to recover the heat of the gases). Failure of both at a time will cause complete failure of the system.

Figure 3 Schematic representation of system



6.1 RAM analysis for system with standby unit

In this section, we are discussing the system when standby unit in E is being taken. The interactions among the various units of the system are shown by RBD in Figure 4.

6.1.1 Data collection

For the RAM analysis, we have taken the data that was extracted from the maintenance reports and it is integrated with the expertise of maintenance personnel. The data was recorded under the following headings:

1 subsystem/facility no.

2 time of onset of failure

3 time to repair

4 type of failure

5 component(s) involved

6 small note describing the cause of the downtime for a period of about nine months.

For each subsystem, the failure rate and repair time of the components are recorded and they are integrated with the expertise of maintenance people. Table 4 presents the summarised data for failure rate and repair time for the components associated with the subsystems.

The interactions among the working components of the system are modelled using PNs and are shown in Figure 5. The minimal cut sets, as obtained by using matrix method, are $\{A_i\}_{i=1,2}$, $\{B_i\}_{i=1,2}$, $\{C\}$,$\{D\}$ and $\{E_1E_2\}$.

Figure 4 Reliability block diagram of the system with standby unit

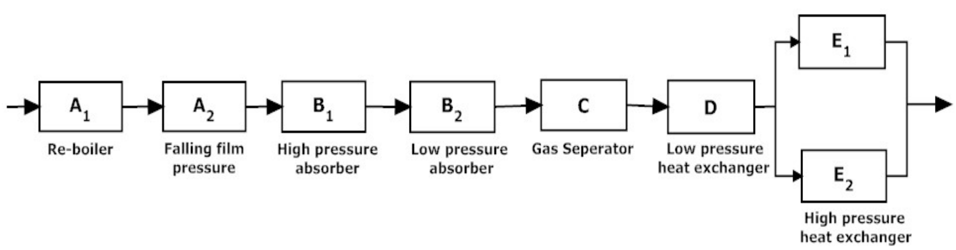
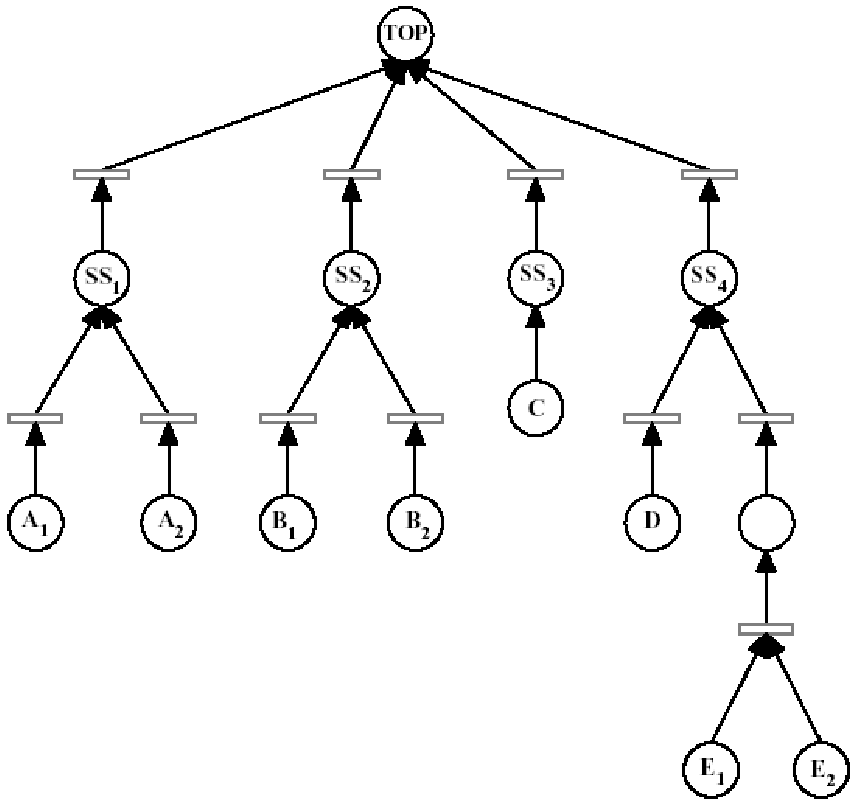


Table 4 Failure and repair statistics of subsystems

Subsystem	SS_1	SS_2	SS_3	SS_4
Failure rate, λ_i (hr^{-1})	4.154	1.592	2.612	6.956
($\times 10^{-4}$) i = 1, 2	3.952	4.783		6.264
Repair time, τ_i (hrs)	3.1746	3.3323	4.899	4.6831
i = 1, 2	2.6421	4.7619		6.2310

Figure 5 PN model of the system with standby unit



6.1.2 Developing a fuzzy number from the reliability data

Reliability data in most databases are reported on a confidence interval basis together with the mean (crisp) value. When such data is reported for a basic event (place), it is simply a matter of representing the triplet of the fuzzy triangular number (a_1, a_2, a_3) with the lower, mean, upper values of the data at 0, 1 and 0 values, respectively. When the lower and upper limits are not given, the analyst may have to choose the lower and upper limits of the fuzzy triangular numbers from his/her engineering experience. In this case, a bias could be introduced by the designer or engineer with regard to the type of use and component operating environment. For this example, we choose a ± 15% spread of the crisp value and the lower and upper limits at 0 membership value and the crisp value at 1.0 membership of the TFN. Other percentage values such as ± 25% and ± 50% were applied to the crisp values to obtain the lower and upper limits of the fuzzy triangular numbers. Use α = 0, 0.1 and 1 with ± 15% spread of the crisp value as the lower and upper limits for the λ and τ values given values, the various intervals of the input fuzzy triangular number corresponding to the α 's can be determined utilising the method of similar triangle and Equation (5) and then calculate the fuzzy triangular number by using Equations (6)–(9) and hence to determine the TOP place utilising the minimal cut set principle.

6.1.3 Determination of reliability parameters

After defuzzifying failure rate and repair time values for the TOP place of the system, they can be used in determining a number of quantifiable parameters such as ENOF, MTBF, etc. of the TOP place. Here, in this example, MTBF, availability, reliability, ENOF were determined for the system's TOP place using the given mission time of 10 hr. The results of the generated minimal cut and path sets together with the quantitative solution can be used in setting up condition monitoring and maintenance planning of the system.

6.1.4 Results

The results of the fuzzy reliability parameters based on various uncertainties on the crisp input and their corresponding lower and upper limits are plotted in Figure 6. The defuzzified values are obtained at ± 15%, ± 25% and ± 50% spreads with crisp values and are tabulated in Table 5 which clearly shows that the crisp values do not change irrespective of the spread chosen. From Table 5, it is evident that defuzzified values changes with change of spread. For instance, failure rate first increases by 0.01%, when spread changes from ± 15% to ± 25%, and further increases by 0.06%, when spread changes from ± 25% to ± 50%. On the other hand, availability decreases by 0.097%, when spread changes from ± 15% to ± 25%, and further decreases by 0.667%, when spread changes from ± 25% to ± 50%. Similar effect is on other one also. Table 6 shows the percentage change in defuzzified values with change of spread for all computed parameters. From this table, it is clear that repair time changes more rapidly, than any other parameters.

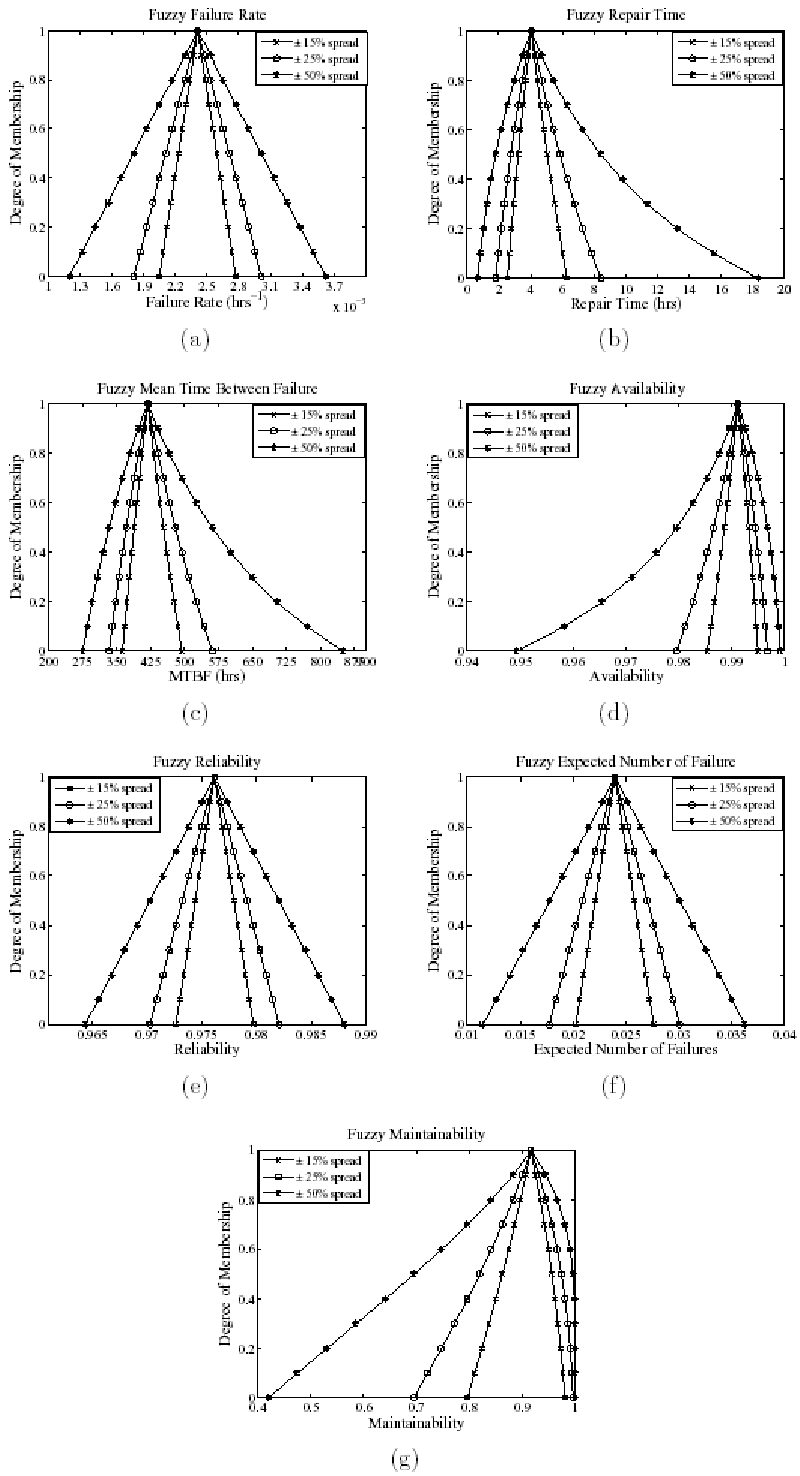
Table 5 Crisp and defuzzified values at different spreads for the system

Reliability parameters	*Crisp value*	*Defuzzified values at spreads*		
		± 15%	*± 25%*	*± 50%*
Failure rate ($\times 10^{-3}$)	2.409790	2.409955	2.410250	2.411630
Repair time	4.034679	4.222529	4.576758	6.690070
ENOF ($\times 10^{-2}$)	2.395099	2.395169	2.395273	2.395625
MTBF ($\times 10^{2}$)	4.190086	4.191681	4.194715	4.213474
Reliability ($\times 10^{-1}$)	9.761901	9.761886	9.761856	9.761722
Availability ($\times 10^{-1}$)	9.911592	9.906692	9.896993	9.831066
Maintainability ($\times 10^{-1}$)	9.161321	9.020572	8.792502	7.988234

Table 6 Change in values of reliability parameters with change in spreads

Reliability parameters	*Change in values (in %)*			*Trends*
	± 0% to ± 15%	*± 15% to ± 25%*	*± 25% to ± 50%*	
Failure rate	0.006855	0.012241	0.057255	Increasing
Repair time	4.655894	8.389025	46.174869	
ENOF	0.002923	0.004342	0.014696	
MTBF	0.038066	0.072381	0.447206	
Reliability	0.000154	0.000307	0.001373	Decreasing
Availability	0.049437	0.097904	0.666132	
Maintainability	1.536340	2.528332	9.147203	

Figure 6 Reliability indices at ± 15%, ± 25% and ± 50% spread with standby unit



To analyse the impact of change in values of reliability indices on to the system's behaviour, behavioural plots have been plotted for different combination of reliability indices and they are shown in Figure 7. In this, for example, Figure 7(a)–(c), two parameters reliability and availability has been fixed to 0.8298 and 0.9203, respectively. Repair time and ENOF have been varied along *x*- and *y*-axes, respectively. The failure rate changed from 0.0022048 to 0.0024098 and further to 0.0027713, the corresponding variation in MTBF along *z*-axis has been computed and plotted. The plots show that as the failure rate of the system increases then for the pre-defined ranges and values of the other indices, MTBF of the system decreases from 4516.55 to 3304.32, 3842.72 to 2809.73 and further from 3344.69 to 2444.17. This suggests that a slight change in system's failure rate may change its MTBF largely and consequently behaviour of the system. Similarly, for other combinations behaviour has been analysed and corresponding results have been shown in Figure 7(a)–(i). These plots will be beneficial for a plant maintenance engineer for preserving particular index and to analyse the impact of other indices on to the system's behaviour.

Figure 7 Behaviour analysis plot for different combination of reliability indices with standby unit

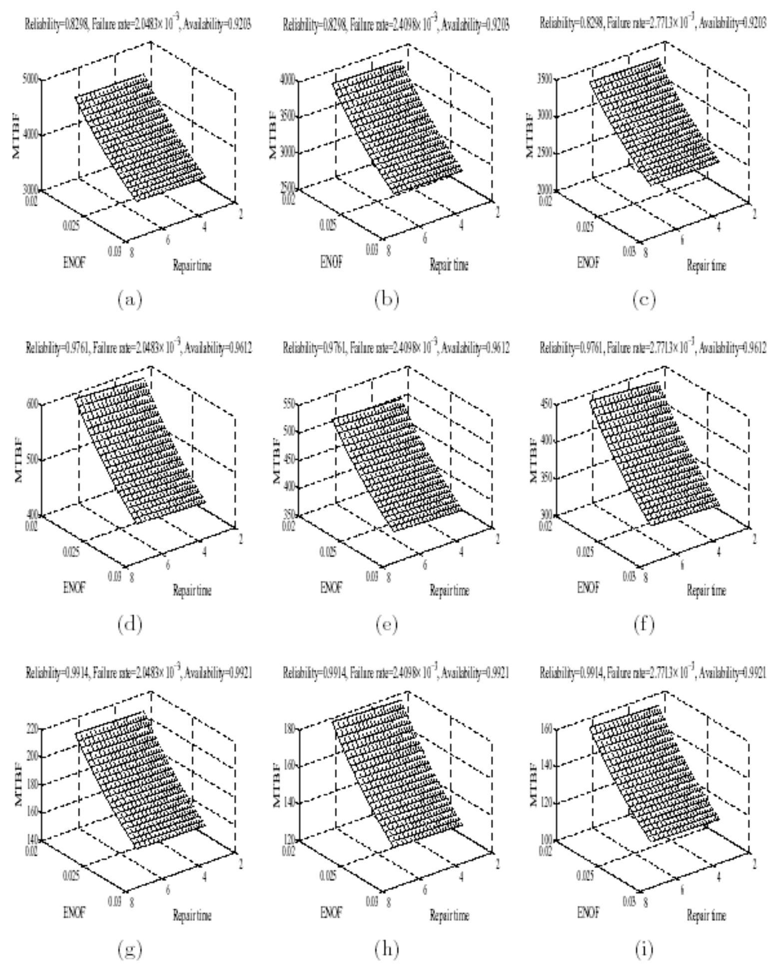
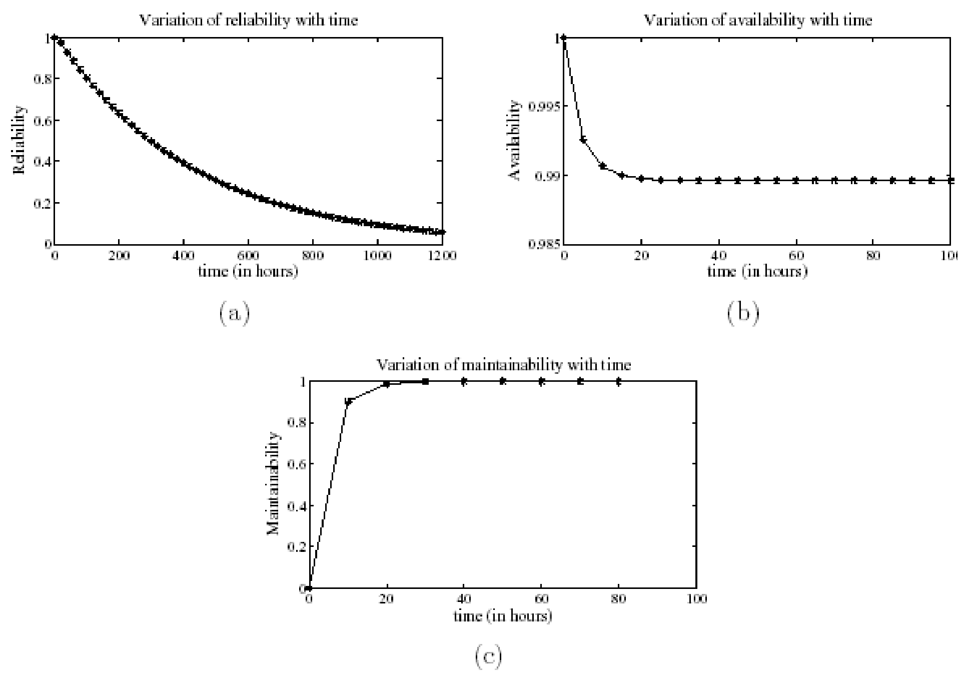


Figure 8 shows the behaviour of the RAM when the mission time varies from 0 to 200 hr and so on. From there, it is clearly seen that the reliability of a system decreases as time passes and becomes zero after a certain time and corresponding maintainability will increase and ultimately attains the value 1.

Figure 8 RAM variation at different mission of time with standby unit



6.2 *When no standby unit is taken*

The RBD for the case when there is no standby unit in E is shown in Figure 9 and the interactions among the working components of the system as modelled using PNs are shown in Figure 10. The minimal cut sets, as obtained by using matrix method, are $\{A_i\}_{i=1,2}$, $\{B_i\}_{i=1,2}$, $\{C\}$,$\{D\}$ and $\{E\}$.

Following the procedures as described in Sections 6.1.2 and 6.1.3, the fuzzy reliability parameters with left and right spreads for the above system are obtained and depicted graphically in Figure 11.

The defuzzified values are obtained at ± 15%, ± 25% and ± 50% spreads with crisp values and are shown in Table 7, which clearly shows that the crisp values do not change irrespective of the spread chosen but defuzzified values change with change of spread.

The percentage changes in defuzzified values with change of spread for all computed parameters are shown in Table 8. For instance, repair time first increases by 8.29%, when spread changes from $\pm 15\%$ to $\pm 25\%$, and Further increases by 45.43%, when spread changes from $\pm 25\%$ to $\pm 50\%$. On the other hand, availability decreases by 0.13%, when spread changes from $\pm 15\%$ to $\pm 25\%$, and Further decreases by 0.88%, when spread changes from $\pm 25\%$ to $\pm 50\%$. Effects on the other parameters can similarly be seen through the Table 8. Sensitivity analysis has been done for various combinations of reliability, availability and failure rate and the effects on MTBF are shown graphically in Figure 12.

The behaviour of the RAM when the mission time is varied from 0 to 200 hr and so on is shown in Figure 13. From there, it is clearly seen that the reliability of a system decreases as time passes and becomes zero after a certain time and corresponding maintainability will increase and ultimately approaches to 1.

Figure 9 Reliability block diagram of the system without standby unit

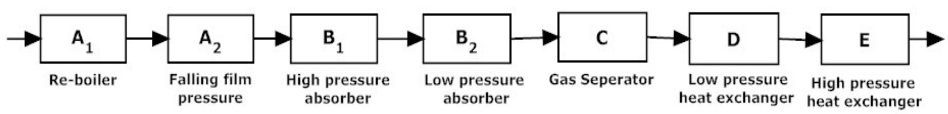


Figure 10 PN model of a system without standby unit

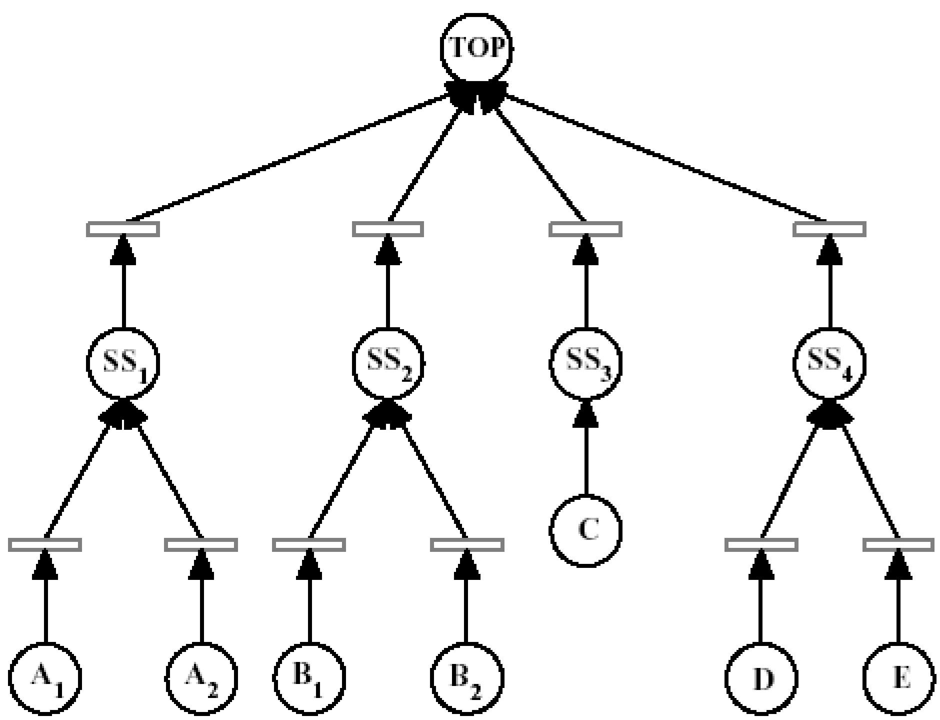


Figure 11 Reliability indices at ± 15%, ± 25% and ± 50% spread without standby unit

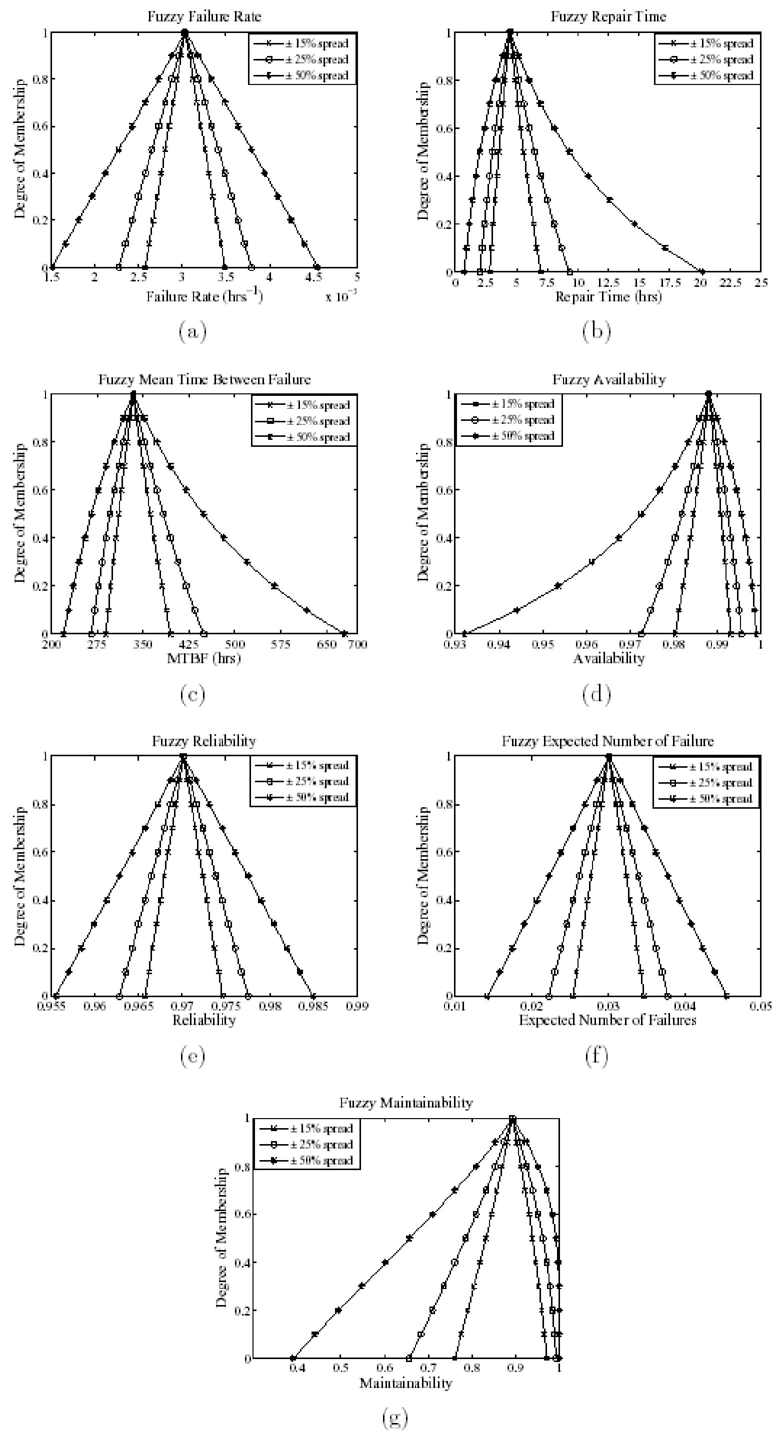


Table 7 Crisp and defuzzified values at different spreads for the system

Reliability parameters	*Crisp value*	*Defuzzified values at spreads* ± 15%	± 25%	± 50%
Failure rate ($\times 10^{-3}$)	3.031300	3.031300	3.031300	3.031300
Repair time	4.490018	4.696914	5.086575	7.397565
ENOF ($\times 10^{-2}$)	3.006741	3.006760	3.006721	3.005930
MTBF ($\times 10^{2}$)	3.343815	3.383839	3.458973	3.899542
Reliability ($\times 10^{-1}$)	9.701418	9.701418	9.701418	9.701418
Availability ($\times 10^{-1}$)	9.879769	9.873277	9.860375	9.772760
Maintainability ($\times 10^{-1}$)	8.921660	8.789709	8.574621	7.815525

Table 8 Change in values of reliability parameters with change in spreads

Reliability parameters	*Change in values (in %)* ± 0% to ± 15%	± 15% to ± 25%	± 25% to ± 50%	*Trends*
Failure rate	No change	No change	No change	Increasing
Repair time	4.607910	8.296107	45.433125	
MTBF	1.196956	2.220378	12.736989	
Reliability	No change	No change	No change	
ENOF	0.000632	0.001297	0.026308	Decreasing
Availability	0.065710	0.130676	0.888556	
Maintainability	1.478996	2.447043	8.852823	

Figure 12 Behaviour analysis plot for different combination of reliability indices without standby unit

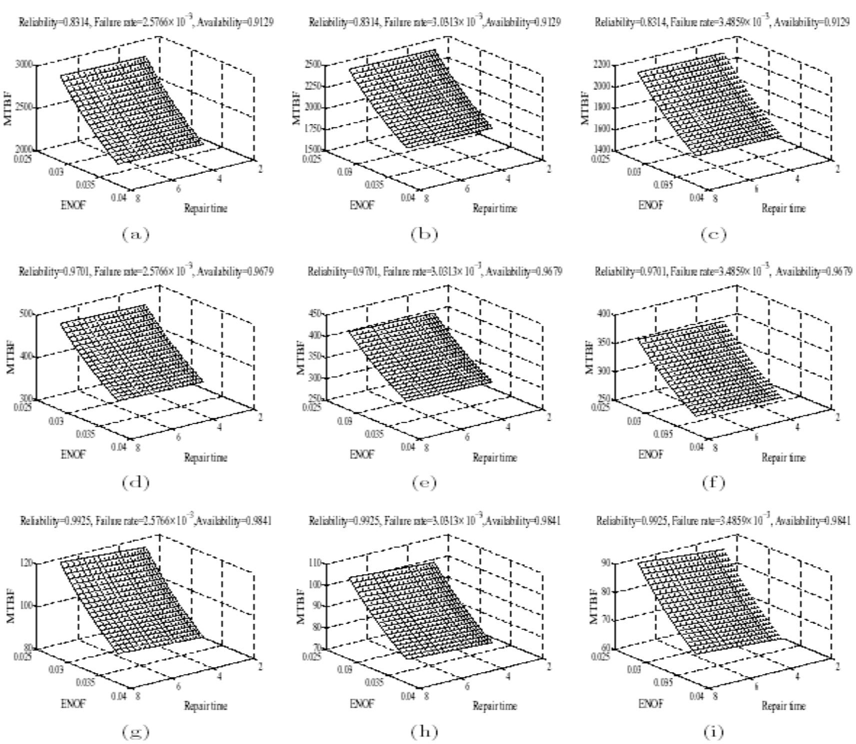
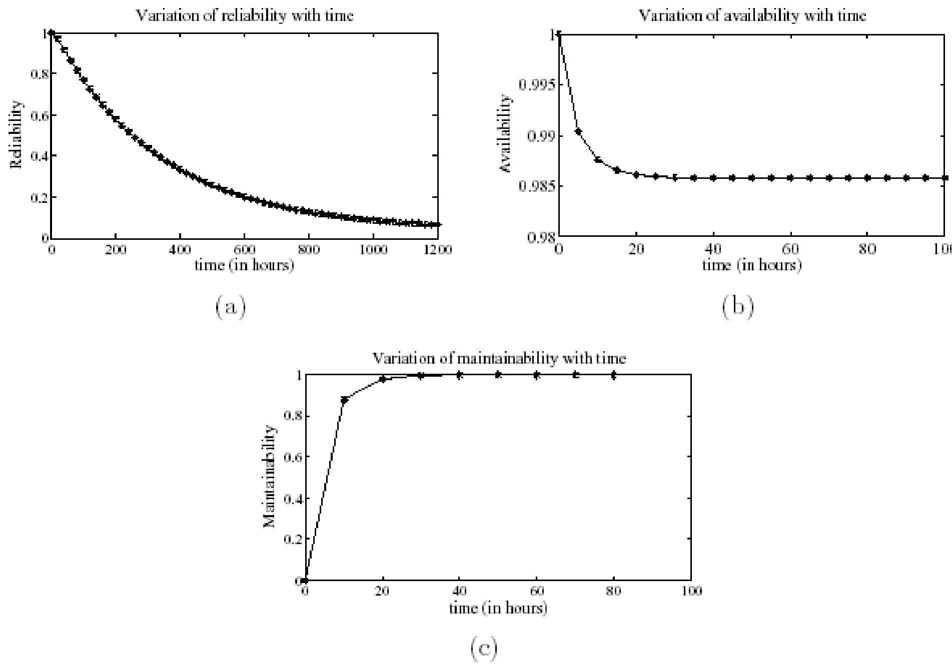


Figure 13 RAM variation at different mission of time without standby unit



7 Conclusion

The development of fuzzy number from available data or components and using fuzzy possibility theory to define membership function can greatly increase the relevance of the reliability study. The use of the fuzzy arithmetic in the PN model increases the flexibility for application to various systems and conditions. This fuzzy reliability methodology has important implications with respect to plant maintenance and operation. It is observed that more appropriately, the maintenance should be performed any time according to the defuzzified MTBF, as a safe interval between maintenance is established. Similarly, it can be realised that with increasing repair time, the reduced value of the reliability/ availability is more conservative than that of the crisp value.

Sensitivity analysis has also been done for various combinations of (R, λ, A) and the effects on system MTBF have been sown graphically. The repair time and ENOF are plotted against x- and y-axes, respectively, whereas MTBF varies along z-axis. The behaviour of MTBF, shown in figures, is summarised in terms of minimum value, maximum value and range of MTBF. From the figures and tables, it is clear that system MTBF changes with change of (R, λ, A) and it is decreases with increase of λ. Based on the behavioural plots and summary given in the tabular form, the system manager can analyse the critical behaviour of the system and may timely plan the suitable maintenance.

The overall benefits for the methodology, however, include

1. the ability to model and deal with highly complex system (because fuzzy sets can deal easily with approximations)
2. the ability to model systems involving multiple experts
3. improve handling of uncertainties and possibilities.

The most important benefit of all is, one can obtain both the crisp, fuzzy and the defuzzified results for even the most highly complex integrated systems with few data.

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